Lesson 3: Arithmetic and Geometric Sequences

Identify each of the following sequences as either arithmetic or geometric write the sequence in the columns below.

|  |  |
| --- | --- |
| Arithmetic | Geometric |
| 1.2.3. | 1.2.3. |

For each arithmetic sequence write the recursive formula and explicit formula

|  |  |
| --- | --- |
| **Recursive****an+1 = an + d** | **Explicit****an = a1 + (n - 1)d** |
| 1. |  |
| 2. |  |
| 3. |  |

For each geometric sequence write the recursive formula and explicit formula

|  |  |
| --- | --- |
| **Recursive****an+1 = an · r** | **Explicit****an = a1(r)n – 1** |
| 1. |  |
| 2. |  |
| 3. |  |

1. Find an explicit form in terms of ***a*** for each of the following arithmetic sequences (assume $a$ is some real number):

$$a, 2a+1, 3a+2, 4a+3,...$$

1. $3+x$**,** $9+3x$**,** $13+4x$**,...** is an arithmetic sequence for some real number $x$.
	1. Find the value of $x$.
	2. Find the 10th term of the sequence.
2. Find an explicit form of the arithmetic sequence where the 2nd term is $25$ and the sum of the 3rd term and 4th term is $86$**.**
3. Consider the arithmetic sequence $ 13, 24, 35,$ ....

a. Find an explicit form for the b. Find the 40th term c. If the $n$th term is $299$, find the value of $n$.

 sequence in terms of ***n***.

1. If $-2, a, b, c, 14$ forms an arithmetic sequence, find the values of $a$, $b$, and $c$.

6. The first term in a geometric sequence is $54$, and the 5th term is $\frac{2}{3}$. Find an explicit form for the geometric sequence.

Lesson Summary

Two types of sequences were studied:

**ARITHMETIC SEQUENCE** (description). A sequence is called *arithmetic* if there is a real number $d$ such that each term in the sequence is the sum of the previous term and $d$.

**GEOMETRIC SEQUENCE** (description). A sequence is called *geometric* if there is a real number $r$ such that each term in the sequence is a product of the previous term and $r$.

Problem Set

For problems 1–4, list the first five terms of each sequence, and identify them as arithmetic or geometric.

1. $A\left(n+1\right)= A\left(n\right)+4$ for $n\geq 1$ and $A\left(1\right)= -2$
2. $A\left(n+1\right)=\frac{1}{4}∙A\left(n\right)$ for $n\geq 1$ and $A\left(1\right)= 8$
3. $A\left(n+1\right)= A\left(n\right)-19$ for $n\geq 1$ and $A\left(1\right)= -6$
4. $A\left(n+1\right)=\frac{2}{3}A\left(n\right)$ for $n\geq 1$ and $A\left(1\right)= 6$

For problems 5–8, identify the sequence as arithmetic or geometric, and write a recursive formula for the sequence. Be sure to identify your starting value.

1. $14, 21, 28, 35,$ …
2. $4, 40, 400, 4000,$ …
3. $49, 7, $ $\frac{1}{7}, \frac{1}{49},$ …
4. $-101, -91, -81, -71,$ …
5. The local football team won the championship several years ago, and since then, ticket prices have been increasing $20 per year. The year they won the championship, tickets were $50. Is the sequence arithmetic or geometric? Write a recursive formula for a sequence that will model ticket prices.
6. A radioactive substance decreases in the amount of grams by one third each year. If the starting amount of the substance in a rock is $1,452$ g, is the sequence arithmetic or geometric? Write a recursive formula for a sequence that models the amount of the substance left after the end of each year.
7. Find an explicit form for each of the following arithmetic sequences (assume $a$ is some real number and $x$ is some real number):

a. $\frac{1}{5}, \frac{1}{10}, 0, -\frac{1}{10},$ ... b. $x+4, x+8, x+12, x+16,…$

1. Consider the arithmetic sequence $ -34, -22, -10, 2,… $ ....

a. Find an explicit form for the b. Find the 20th term c. If the $n$th term is $434$, find the value of $n$.

 sequence in terms of ***n***.

1. Find the common ratio and an explicit form in each of the following geometric sequences:
	1. $4, 12, 36, 108,$ ... b. $162, 108, 72, 48, $... c. $\frac{4}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{6},$ ... d. $xz, x^{2}z^{3}, x^{3}z^{5}, x^{4}z^{7},$ ...

$-2, 2, 6, 10,$ … $2, 4, 8, 16,. . . $

$\frac{1}{2}, 1, \frac{3}{2}, 2,$ $\frac{5}{2},$ … $1,$ $\frac{1}{3}, \frac{1}{9}, \frac{1}{27},$ …

$10, 1, 0.1, 0.01, 0.001,$… $4, -1, -6, -11,…$